



# Pricing and production in consumer markets where sales depend on production



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## HIGHLIGHTS

- A market where sale depends on output and rationing is proportional is studied.
- Identical firms prefer a market price which maximizes the industry's revenues.
- Revenue maximization is also preferred by small firms, not necessarily identical.
- If all firms prefer the same market price, price leadership will realize this price.
- Price leadership is both possible and legal in consumer markets.

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## ABSTRACT

This paper deals with consumer markets where sales depend on production. Assuming that a firm's sale is proportional to its production, it shows that a price leader will set a price which maximizes its industry's revenues in markets with many small firms, independent of their cost functions.

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## 1. Introduction

In markets where production precedes sales producers have to 'bring their goods to a market place' before sales are completed. The traditional approach to this problem in oligopoly theory is based on the Cournot model, where firms choose quantities and a market-clearing auctioneer or process sets the market price. This is also an adequate model in markets for agricultural products and oil and other raw materials, where an auctioneer sets that price which equals demand to that supply which has been brought to the market place. But it is not applicable to markets where buyers take prices as given and prices consequently are set by sellers, as in most consumer markets.

A strictly non-cooperative approach to pricing by firms corresponds to sealed bidding. However, while sealed bidding may

be enforced by a big buyer, as in construction or in sales to the public sector, it is not applicable to markets where buyers take prices as given and prices are set by firms. Of course, in such markets prices are set by firms before trade can start. But firms can also observe and revise their prices and establish a market price at which all firms can trade and share the market, in contrast to sealed bidding, where only the firm with the lowest bid will trade.

In this case *price leadership* is the appropriate market form, as suggested by Boulding (1941, pp. 607–613) and argued in more detail in Farm (forthcoming). Price leadership means that all firms but one take the price as given or, more precisely, that one of the firms sets a price which the other firms match. And while setting the same price as another firm suggests collusion in markets with sealed bidding, it is both possible and legal in markets where firms are free to observe and revise their prices at any time, as in most consumer markets. Moreover, if firms prefer different market prices, due to differences in costs, capacities, or market shares, then a firm preferring the lowest market price can enforce this price and

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become a *competitive* price leader simply by setting it. In terms of price adjustment, the market price goes down if and only if a price cut appears profitable to a firm even if its competitors follow suit.

In this paper, however, all firms will prefer the same market price. Then the choice of price leader is immaterial and may be expected to vary randomly or depend on which firm is assumed to have the best information on market conditions. A price leader may in this case be called a *barometric* price leader, following [Stigler \(1947\)](#).

A firm often produces only what it can sell at the price it sets. This is always true if sales precede production (production to orders) and often true when production precedes sales (production to stock). More precisely, with production to stock, as in most consumer markets (where customers usually have to visit shops to find what they want to buy), a firm can often anticipate its sales at the price it sets. Of course, production will in general differ from sales and the difference will change the firm's inventories. But such changes are often negligible, so that output adjusts to sales, at least approximately, even in markets where production precedes sales, provided that sales are independent of production, as in [Farm \(forthcoming\)](#).

This paper, however, considers the case when sales do depend on production. In this case, and when rationing is proportional, we shall see that all firms in an industry prefer a market price which maximizes the industry's revenues, not only when firms are identical but also when there are many small firms, independent of their cost functions. It follows that a (barometric) price leader will set the revenue-maximizing price. And price leadership is not only possible but also legal in markets where firms are free to observe and revise their prices at any time, as in most consumer markets.

## 2. Choice of production by price-taking firms

Consider first the determination of production at different market prices. Firms' outputs at a given market price are determined exclusively by supply only at the market-clearing price. At higher prices they will also depend on consumer choice and marketing (but we exclude the possibility for firms to fix market shares).

To see what marketing implies for price leadership, define aggregate supply at the market price  $p$  in the usual way as  $S(p) = \sum s_j(p)$ , where the supply  $s_j(p)$  of firm  $j$  is derived on the presumption that everything produced will be sold. This presumption is also true if  $p = p^c$ , where  $p^c$  clears the market,  $D(p^c) = S(p^c)$ , where  $D(p)$  denotes demand at  $p$ . But it is not true if  $p > p^c$ . Producers should realize this and adjust production accordingly—assuming that they do take the market price  $p$  as given when production is determined.

At this stage market sharing has to be specified. Now, a firm's output can either influence its sales or not. If not, a firm will adjust its production to its sales, and market shares will be determined as in markets with production to orders, as elaborated in [Farm \(forthcoming\)](#). On the other hand, market shares can also be influenced by making goods available to consumers in shops. And assuming that availability in shops is proportional to output distributed among shops in the market, a firm's market share will be

$$\alpha_i = q_i / \sum q_j, \quad (1)$$

where  $q_i$  denotes a firm's production. This is *proportional rationing*, where every unit of supply of a good has the same probability of being sold in the market.

It follows that a firm's profit function is

$$\pi_i = pD(p)q_i / \sum q_j - c_i(q_i), \quad (2)$$

where  $c_i(\cdot)$  denotes a firm's cost function, assuming in addition (for simplicity) that output remaining at the end of the market period is without value.<sup>1</sup> Differentiation yields

$$\frac{\partial \pi_i}{\partial q_i} = p(1 - \alpha_i)d - c'_i(q_i), \quad \text{where} \quad (3)$$

$$d = D(p) / \sum q_j. \quad (4)$$

It follows that  $(q_i)$  is a self-enforcing agreement if

$$p(1 - \alpha_i)d = c'_i(q_i) \quad \text{for every } i. \quad (5)$$

## 3. Production and pricing in consumer markets with identical firms<sup>2</sup>

Next we consider an industry with  $n$  identical firms producing at constant returns with the marginal cost  $c$  when the market price is  $p$ . In this case (5) reduces to the system of equations  $p(1 - \alpha_i)d = c$ , which is solved by  $\alpha_i = 1/n$  and  $d = (c/p)(1 - 1/n)$  if  $d \leq 1$  or, equivalently,  $p \geq c/(1 - 1/n)$ . Since  $q_i = D(p)/nd$  according to (4), firms taking the market price  $p$  as given will consequently in equilibrium restrict production to

$$q_i^e(p) = (1 - 1/n)(p/c)D(p)/n \quad \text{if } p \geq c(1 - 1/n). \quad (6)$$

Moreover, it follows from  $\pi_i = (pd - c)q_i$ ,  $d = (c/p)/(1 - 1/n)$  and (6) that a firm's profits in equilibrium after quantity adjustment at the market price  $p$  will be

$$\pi_i^e(p) = pD(p)/n^2 \quad \text{if } p \geq c/(1 - 1/n). \quad (7)$$

If on the other hand  $p < c/(1 - 1/n)$ , then  $q_i = D(p)/n$  is an equilibrium point, since for  $\alpha_i = 1/n$  and  $d = 1$  we have  $\partial \pi_i / \partial q_i = p(1 - \alpha_i)d - c = p(1 - 1/n) - c < 0$ , implying that

$$q_i^e(p) = D(p)/n \quad \text{and} \quad \pi_i^e(p) = (p - c)D(p)/n \quad \text{if } p < c/(1 - 1/n). \quad (8)$$

Assuming that  $(p - c)D(p)$  is increasing in  $p$  up to  $p^m$ , it follows that every firm prefers  $p^0 = \arg \max pD(p)$  as the market price if demand is sufficiently inelastic,  $p^0 \geq c/(1 - 1/n)$ , since  $p^0 < p^m$  and  $\pi_i^e(p)$  is continuous at  $p = c/(1 - 1/n)$ . This completes the proof of the following result:

**Proposition 1.** *Consider a market where production precedes sales, rationing is proportional and there are  $n$  firms producing at constant returns with the same marginal cost  $c$ . Then all firms prefer the same market price, namely  $p^0 = \arg \max pD(p)$ , if demand is sufficiently inelastic,  $p^0 \geq c/(1 - 1/n)$ .*

Thus, costly competition for market shares through non-cooperatively chosen production will make it profitable for a price leader in an industry with identical firms to set a price which is lower than the monopoly price, namely that price which maximizes the industry's revenues. Note that, according to (7), new firms would reduce profits for incumbents not only at the rate of  $1/n$ , because of more firms sharing the same revenues, but at the rate of  $1/n^2$ , because of additional supply in equilibrium. Note also that the case of elastic demand is not particularly interesting, since then not even a monopolist would set a high price.

<sup>1</sup> Note also that stocks remaining at the end of the market period are often sold at a reduced price (or simply scrapped). In any case, adding an inventory evaluation function will not change the substance of the analysis.

<sup>2</sup> This section draws on [Farm \(1988\)](#), where, however, price leadership is not even mentioned.

**4. Production and pricing in consumer markets with small firms**

We shall now see that the result above for identical firms can be generalized to atomistic markets, irrespective of the cost functions, provided that firms are so small that we can set  $\alpha_i = 0$  in (5). For then it follows that  $pd = c'_i(q_i)$  or, equivalently,  $q_i = s_i(pd)$ . Hence  $\sum q_i = S(pd)$  and  $d = D(p)/S(pd)$  so that  $d = d(p)$  solves the equation

$$D(p) = S(pd) d. \tag{9}$$

Assuming, as we always do in this paper, that  $D(p)$  is decreasing in  $p$  and  $S(p)$  increasing or constant, the solution to this equation is unique, and then we have the following result:

**Proposition 2.** *In a market where production precedes sales and rationing is proportional, ‘small’ firms taking a market price  $p > p^c$  as given will produce  $q_i^e(p) = s_i(pd(p))$  in equilibrium, where  $d(p)$  is defined implicitly by (9).*

Note that  $d(p) < 1$  for  $p > p^c$  so that  $q_i^e(p) = s_i(pd(p))$  is indeed an interior solution (excluding market clearing) and  $1 - d(p)$  is the equilibrium rate of excess supply. Also note that  $pD(p) = S(pd(p))pd(p)$ , so that, with our assumptions on demand and supply, it follows from  $(pD)' = S'(pd)(pd)'pd + S(pd)(pd)'$  that

$$\text{sign}(pd(p))' = \text{sign}(pD(p))'. \tag{10}$$

Hence the equilibrium supply curve

$$Q^e = Q^e(p) = S(pd(p)) \tag{11}$$

is backward-bending at  $p$  if the usual potential supply curve  $S(p)$  is forward-bending and demand is elastic,  $-pD'(p)/D(p) = \eta(p) > 1$ , since  $(pD)' = D(1 - \eta)$ . Thus, if demand is elastic for every  $p > p^c$ , equilibrium supply will be less than  $D(p^c)$  for every  $p > p^c$ . And if demand is inelastic at  $p^c$ , equilibrium supply will be increasing up to  $p^0 = \arg \max pD(p)$  and then decreasing. Now we can prove that every firm prefers the same market price, irrespective of its cost function:

**Proposition 3.** *In a market where production precedes sales, rationing is proportional, and firms are ‘small’, all firms prefer the same market, namely  $\max(p^c, p^0)$ , where  $p^c$  is defined implicitly by  $D(p^c) = S(p^c)$  and  $p^0 = \arg \max pD(p)$ .*

**Proof.** Recall that a firm’s profit in equilibrium after quantity adjustment is

$$\pi_i^e(p) = pd(p)q_i^e - c_i(q_i^e), \tag{12}$$

where  $d(p)$  is defined by (9) and  $q_i^e = s_i(pd(p))$  maximizes  $\pi_i = pd(p)q_i - c_i(q_i)$ . It follows from the envelope theorem that

$$\frac{d\pi_i^e(p)}{dp} = \frac{\partial (pd(p))}{\partial p} s_i(pd(p)), \tag{13}$$

and hence that  $\pi_i^e(p)$  is maximized by  $\arg \max pd(p)$ , which is equal to  $\arg \max pD(p)$  according to (10).

To complete the model it is hardly realistic to assume that one of the small firms is a price leader. Instead we assume, as in the traditional model of perfect competition, that all firms are price takers. But since I here stick to the price-taking postulate, I also assume, in order to model an orderly market, that the industry has a trade association which sets the market price but cannot restrict production. And realizing that every firm prefers that market price which maximizes the industry’s revenues, the trade association’s problem is that of a statistician, namely to estimate the demand function and especially its price elasticity.

**5. Concluding comments**

Is this model of a market with ‘many’ firms applicable to any real market? Grain farming is an industry with many firms, but then one can hardly assume that buyers take prices as given so that prices are completely determined by firms. (Instead prices are set in auction markets, including markets for futures, often also including intervention by a government.) But a taxi market is probably a relevant example (provided that customers take prices as given). For instance, deregulation of a taxi market which introduces free entry will not lower the market price much but increase the number of cabs, according to the model above, so there will be some excess supply in equilibrium (if product demand is sufficiently inelastic).

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