Labour Demand and Product Demand*

Ante Farm

ante.farm@sofi.su.se

Swedish Institute for Social Research (SOFI), Stockholm University

SE-106 91 Stockholm, Sweden

www.sofi.su.se

October 24, 2017

Abstract: This paper shows that the traditional model of labour demand in a firm is incomplete and that the determinants of labour demand in a complete model include the demand for the firm's output.

Keywords: Labour demand, employment

JEL-Code: J23

* I thank participants of seminars at SOFI, and in particular Jim Albrecht, Anders Björklund, Susan Vroman, and Eskil Wadensjö, for useful comments on earlier versions. A comment by Per Lundborg during the writing of the present version has also been very useful.

1. Introduction

What are the determinants of employment in the short run, when the capital stock is given? The most well-known proposition in this field is that a competitive firm chooses its level of employment by setting the value of the marginal product equal to the wage,

(1) $$pF'(N) = w \text{ if } F'' < 0,$$

where N denotes employment (in hours), w the wage, p the product price, and F the production function. And the corresponding result for a monopoly is

(2) $$(1 - 1/\eta)pF'(N) = w \text{ if } F'' < 0,$$

where η denotes the price elasticity of the firm's product demand.[1]

Both (1) and (2) suggest that labour demand only indirectly depends on the level of product demand, which makes it difficult, for example, to explain the transmission of product-demand shocks to the labour market, as emphasized, for example, by Lindbeck (1998). Of course, (2) suggests that employment depends on product demand through its price elasticity, as also noted by Hamermesh (1993 p. 22), but since $p = D^{-1}(F(N))$ the relation between product demand and employment is not particularly transparent. And according to (1) the labour demand of a competitive firm does not depend on product demand at all, only on the production function and the real wage (w/p).

On the other hand, as emphasized in (almost) every textbook in economics, labour demand is a derived demand. The dependence of labour demand on product demand through a price related to costs is also emphasized in most textbooks *at the industry level*, for example in Borjas (2008 p. 131), when Marshall's laws of derived demand are discussed. And if labour demand is a derived demand at the industry level, it should also be a derived demand at the firm level. The purpose of this paper is to show that this is indeed the case.

The intuition is as follows. The traditional motivation for (1) and (2) is that a profit-maximizing firm increases employment until the value of the marginal product of labour is equal to the wage rate. However, in practice a firm does not chose employment but the price of its output before trade can start. And then it adjusts its production to its sales and its employment to its production. Since neither (1) nor (2) reflect these facts, they cannot be complete.

To model the problem formally we begin by looking at a monopoly in Sections 2 and 3, followed by a reappraisal of the traditional approach to labour demand for a competitive firm

[1] See any introductory textbook in economics or labour economics but also, for instance, Hamermesh (1993 p. 22) and Cahuc and Zylberberg (2004 p. 175).

in Section 4. In both cases we find that the traditional model is incomplete and that a complete model includes product demand as one of the determinants of labour demand. The resultant model of a firm's employment is then generalized to include intermediate goods in Section 5 and recruitment costs in Section 6. Section 7 concludes.

Throughout the paper we study labour demand as usually defined, i.e. the relation between employment and its determinants on the assumption that firms can hire all the labour they want. We also assume that firms are free to adjust prices and employment to wages once wages have been set, with or without bargaining.

2. Labour demand of a monopoly

As emphasized by Layard, Nickell and Jackman (1991 p. 341), eq. (2) is an equilibrium relationship: "It is not a labour demand function because prices are chosen jointly with employment". Thus, in a market with monopoly, and assuming only one input to begin with,[2] a firm's complete problem is

(3) $$\max_{p,N} pF(N) - wN \text{ s.t. } F(N) = D(p),$$

where $D(p)$ denotes the firm's sales at the price *p*. And this problem is solved by the three equations $F(N) + \lambda D'(p) = 0$ $\;pF'(N) - w - \lambda F'(N) = 0$ and $F(N) = D(p)$, where λ is the Lagrange multiplier. Hence

(4) $$\lambda = \frac{pF'(N) - w}{F'(N)} = -\frac{D(p)}{D'(p)}, \text{ so that } \frac{pF'(N) - w}{F'(N)} = \frac{p}{\eta},$$

where $\eta = -pD'(p)/D(p)$. It follows that *p* and *N* are determined by the two equations

(5) $$p = \mu w / F'(N),$$

(6) $$F(N) = D(p),$$

where $\mu = 1/(1 - 1/\eta)$. Thus, the fundamental problem with (5) is not that it is false but that it only indirectly affects employment, through its effect on the product price, while the direct determinants of employment are incorporated in (6). Of course, this solution can also be obtained by first maximizing $pD(p) - wF^{-1}(D(p))$ with respect to *p* and then setting $N = F^{-1}(D(p))$.

[2] A model with labour as the only input is not so unrealistic as it seems, since *w* can be interpreted as total variable costs per hour, not only labour costs but also costs for intermediate goods used in the production process, as elaborated in Section 5.

It follows that (5) or, equivalently, (2) should be interpreted not as a labour-demand function but as a *price equation* – as also emphasized in macroeconomic literature based on the price-setting curve (e.g. Layard et al. 1991). This is particularly clear when $F''(N)=0$ and $F'(N)=a$, since then $p=\mu w/a$ according to (5) – and $N=D(p)/a$ according to (6).

Note also that the classical model of a monopoly focuses on the pricing problem,

$$\max_p pD(p)-cD(p), \tag{7}$$

where *c* is (constant) marginal cost. This problem is solved by the solution to the equation

$$(p-c)/p=1/\eta(p), \tag{8}$$

assuming (implicitly) that while the firm sets the price, its customers determine its sales, and also assuming (implicitly) that the firm adjusts its production (and the corresponding employment) to its sales. Thus, for a monopolist the basic decision variable is its price, but the firm also has to adjust output to sales and employment to output. Choosing employment simultaneously with price presupposes, of course, perfect information on product demand.

3. A special case

Let us next discuss a special case which also can be interpreted as a useful first approximation. Suppose that the marginal productivity is constant, $F'(N)=a$, up to a certain employment level equal to $\hat{N}$, where it begins to fall. If the fall is very strong, then output cannot be much higher than $a\hat{N}=k$, which consequently characterizes the firm's *capacity*. This example is not only useful as a bench-mark but also rather realistic, as argued, for example, by Layard et al. (1991 p. 340).[3]

Of course, a firm's supply curve is not vertical for high prices even if it usually is rather steep due to constraints on employment in existing premises and with existing machinery (and restrictions on overtime etc.). Capacity is consequently in general not a parameter but an increasing function of the price. However, assuming a constant capacity simplifies the analysis considerably without changing its substance.

To initiate sales the firm has to announce a price in a market where buyers take prices as given, as in most consumer markets. If the firm anticipates that product demand will be low in relation to its capacity, it will announce $p=\mu w/a$. This formula shows how the firm adjusts

[3] See also, for example, Blinder et al. (1998 p. 102) on the prevalence of constant marginal cost.

its price to changes in wages and productivity for a given mark-up, while the mark-up is adjusted by the firm according to its perceptions of the price elasticity of product demand. And the firm adjusts its employment according to (6) as long as $D(\mu w/a) \leq k$. Thus,

(9) $$p = \mu w/a \text{ and } N = D(p)/a \text{ if } D(p) \leq k,$$

while the real wage (w/p) is determined by labour productivity and mark-up, $w/p = a/\mu$.

If on the other hand $D(\mu w/a) > k$, price and employment are adjusted by the firm, assuming perfect information on the demand function, until the equations (5) and (6) are satisfied. When the fall in marginal productivity is very strong for employment above $\hat{N}$, employment is (approximately) equal to $\hat{N}$, while the price is raised until (quantity) rationing has been eliminated. Thus, as a first approximation,

(10) $$p = D^{-1}(k) \text{ and } N = k/a \text{ if } D(\mu w/a) > k.$$

According to (10), variation in product demand will only affect prices but not employment in a boom, if capacity constraints are binding. This is, in general, only a first approximation. But it does represent the reasonable notion that a firm's employment can only increase marginally when its capacity has been reached. Formally, this marginal adjustment involves (5), but (5) only applies to a very small interval of employment, assuming that $F'(N)$ declines rapidly above $\hat{N}$. And during this adjustment of employment the price will also rise, according to (5) and (6).

We conclude that, as a first approximation, employment is *never* determined by (2), at least not directly. Employment is determined *indirectly* by (2) in a recession, since then (2) determines the product price, while employment is determined by the demand for the firm's output at this price and the firm's labour productivity, according to (9). And in a boom (2) has a minor effect on employment, provided that production and employment are restricted by the firm's capacity, according to (10). While (5) and (6) constitute a complete model of price and employment at the firm level, (9) and (10) constitute a useful first approximation when a firm's production function can be characterized by two parameters, its labour productivity (*a*) and its capacity (*k*).

4. Labour demand of a competitive firm

These results for a monopoly generalize easily to a competitive firm, but only if we complete the model by adding the fact that a competitive firm must be part of a competitive industry.

For then we can see that not only (2) but also (1) is incomplete. In fact, price *p* and employment *N* of a firm in a competitive industry with an arbitrary number (*n*) of identical firms with production function *F* and a wage level equal to *w* are determined by the equations

(11) $$p = w/F'(N),$$

(12) $$nF(N) = D(p),$$

where *D* is the industry's product-demand function and $F'' \leq 0$.

In this complete model of a representative competitive firm, eq. (11) models the notion of a price-taking firm which is in equilibrium only when price equals marginal cost, while eq. (12) models the notion that supply equals demand in equilibrium. Eq. (12) also implies that every firm has the same market share, but this assumption is made for simplicity only.

Now, if the marginal productivity is constant, $F'(N) = a$, (11) completely determines price, while (12) determines employment. And this happens if product demand is low, so that

(13) $$p = w/a \text{ and } N = D(p)/na \text{ if } D(p) \leq nk,$$

where *k* denotes a firm's capacity, as in Section 3. Production and employment can consequently be restricted by sales even in a competitive industry. And then an individual firm will also be restricted by sales, or more precisely by its market share, which in our simple example with identical firms is $1/n$. (If firms are identical, and no capacity constraint is binding, the probability that a consumer chooses to buy from a particular firm is $1/n$ if there are *n* firms, and it follows from the law of large numbers that each firm's market share is $1/n$.) Note that this possibility is excluded *by assumption* in the traditional model of employment in a competitive firm in the short run.

Moreover, capacity constraints ($F'' < 0$) will *raise* prices (since $p = w/F'(N) > w/a$) but *reduce* the effect of wage changes on employment. In fact, as a first approximation,

(14) $$p = D^{-1}(nk) \text{ and } N = k/a \text{ if } D(w/a) > nk.$$

In a boom production and employment will consequently be restricted by capacity (if the boom is sufficiently strong) and the market price will be the market-clearing price. And employment will be constant or only marginally affected by (11).

Since (11) or, equivalently, (1) is so firmly established in the literature, it is perhaps hard to accept that it determines employment in a competitive firm only partly and indirectly, through its determination of the market price. The marginal-productivity function is of course a basic determinant of employment in a firm. But it determines employment essentially through two parameters: its labour productivity (a) and its capacity (k). At least this is true as a first approximation. I have also argued that this first approximation is probably rather good, and it is certainly helpful for the intuition. But note that it is not crucial for my argument. Eq. (1) is an incomplete model of employment in a competitive firm even if $F''(N) < 0$ for every N.

Note also that, apart from the mark-up, results are the same for a competitive and a non-competitive firm. This is because I have relaxed an implicit assumption of the traditional model of a competitive market, namely that a price-taking firm can never be restricted by what it can sell. The necessity to relax this assumption is most obvious with constant returns, when production must be restricted by sales in the firm's industry and hence also in the industry's firms.

Now, even if the basic principles of labour demand can be presented with labour as the only input, it remains to make the model completely complete by adding intermediate goods and recruitment activities.

5. Intermediate goods

Suppose that not only employment is proportional to output q, $N = q/a$, but also other variable inputs, $M = q/b$, so that variable costs can be written as

(15) $$C = (w+g)N \text{, where } g = va/b .$$

Thus we assume that even variable costs other than labour costs are proportional to employment, with an addition to the wage rate (g) which depends on the prices of additional inputs (v) as well as the relation between employment and other inputs ($b/a = N/M$).

We also assume, to begin with, that not only a firm's capacity but also its technology is fixed in the short run. Even if a firm in the long run attempts to reduce its marginal cost $c = (w+g)/a$ at anticipated input prices by an appropriate substitution between labour and other variable inputs (like semi-finished goods), it can hardly change this mix instantaneously when input prices are revised. A firm can adjust its output prices almost instantaneously to new input prices, but it takes longer to change its technology.

With these assumptions it follows that

(16) $$p = (1+m)c = (1+m)(w+g)/a \text{ and } N = D(p)/a \text{ if } D(p) < k ,$$

where p denotes the price the firm sets, $D(p)$ the firm's sales at this price, and m is a mark-up which equals 0 for a firm in a competitive market and $1/(\eta-1)$ for a monopoly or, more precisely, $m = e/(1-e)$ where $e = (p^m - c)/p^m$ and p^m solves $(p-c)/p = 1/\eta(p)$.

Thus, a firm's production is restricted either by its sales, according to (16), or by its capacity. And if production is restricted by capacity, then employment is also restricted by capacity, implying that the wage level and other elements of marginal cost have a negligible impact on employment (at least as a first approximation).

Note that the cost of intermediate goods will reduce the wage elasticity of labour demand, in accordance with Marshall's laws of derived demand (Marshall 1982 p. 319). In fact, as differentiation of (16) shows, the wage elasticity of labour demand is equal to the price elasticity of product demand multiplied by labour's share in total variable costs, $w/(w+g)$.

The production technology is characterized by three parameters, namely capacity (k), labour productivity ($a = q/N$) and input technology ($a/b = M/N$). Labour productivity is a summary measure of the real effect of labour, machinery and intermediate goods in the production of output, while input technology (the relation between intermediate goods and employment) measures the firm's dependence on current production in other firms.

Note finally that I have shown how product demand affects labour demand *in the short run*, at given capacities, technologies and skills. If, however, technology, organization or real capital can be changed as a response to a change in wages, we have to distinguish between 'substitution effects' and 'scale effects'.[4] For example, a rise of the wage level may get the firm to substitute intermediate goods or services for some employee-hours in the intermediate run or new machines in the long run. Or a rise of wages for some workers may get the firm to substitute one type of workers for another type. In any case, if a change of the wage level or the wage structure leads to substitutions in some respects, this may have a substantial indirect effect on employment after some time.

In general, and in the long run, substitutions may affect not only intermediate goods (g) and labour productivity (a) but also the average wage (w), implying that the effect on marginal cost ($c = (w+g)/a$) in the long run may differ from the short-run effect of a change in w. In the short run we may have a scale effect from a change in wage level on the product price and hence on production and employment, while substitution effects of various kinds

[4] On the distinction between substitution effects and scale effects related to the choice of labour and capital in the long run, see, for instance, Section 1.2 in Chapter 4 in Cahuc and Zylberberg (2004).

may modify both price and labour productivity and hence also employment in the long run. Note, however, that at every point in time price and employment are determined by (16) with current values of the determinants.

6. Recruitment costs

If a firm expects an increase in the demand for its products to be only temporary, it will not necessarily increase its employment, due to costs of hiring and firing, as elaborated in the literature on adjustment costs.[5] But recruitment costs can reduce employment even in a steady state (where we can ignore firing costs), provided they by raising product prices also reduce demand for the firm's products.

To see how recruitment costs add to variable costs we begin by observing that the variable costs for a firm with employment N in general can be written as

$$C = wN + gN + \alpha H + \gamma V \,, \tag{17}$$

where H denotes the firm's number of hires per period and V its stock of job vacancies. The wage level is denoted by w, other variable production costs are measured by gN, as in Section 5, while recruitment costs are captured by the parameters α, as in Nickell (1986), and γ, as in Pissarides (1990).

Note that recruitment costs are in general composed of both *hiring costs* (α per hire) and *vacancy costs* (γ per job vacancy and period). Hiring costs include costs of introduction and training but also costs of job advertising if these are concentrated to the beginning of the recruitment process, while vacancy costs include recruitment costs which increase with the length of recruitment, like a fee to a private employment agency if the firm is paying the agency for its services per week and not per job match.

Moreover, the stock of job vacancies (V) is related to the flow of hires (H) according to

$$V = fHT, \tag{18}$$

where f denotes the share of hires preceded by job vacancies, so that fH measures the inflow of job vacancies, and T denotes the average duration of job vacancies. The parameter f is included because much hiring is not mediated through job vacancies as measured in vacancy surveys, as shown by Davis, Faberman, and Haltiwanger (2013) for the U.S. This includes in particular "instantaneous" hires, like recalls of former employees.

We assume in this paper – as in the literature on adjustment costs – that *all* hires are instantaneous, including hires preceded by job vacancies. To see why, note first that firms

[5] See, for example, Nickell (1986) and Hamermesh (1993).

create job vacancies (recruitment processes) in order to avoid unfilled jobs (unmet labour demand). Thus, firms start recruiting in anticipation of future needs. And if, for instance, a separation can be anticipated and a replacement made before the separation, then replacement is instantaneous even if recruitment is not.

Note that eq. (17) already excludes unfilled jobs, because employment is assumed to be constant over time in (17). In other words, this paper deals with the effect on employment of costly search on the simplifying assumption that firms completely control employment. This may be a reasonable approach if unfilled jobs are rare and hard to predict, so that firms simply ignore them when prices are adjusted to recruitment costs.[6]

The approach may also be reasonable for employers who anticipate problems to keep employment constant, provided it incorporates plans to use substitutes (including personnel from temporary work agencies) whenever substitutes are necessary during recruitment of replacements in order to avoid unfilled jobs. But then anticipated costs of the necessary substitutes must be added to vacancy costs, even if only vacancy costs *above* the wage level w can be included, since wN in (17) already includes the costs of having posts occupied.

Now, in a steady state, with product demand and other market conditions assumed to be constant for some time, a firm only has to replace separations. Separations occur for a variety of personal or institutional reasons and are not necessarily proportional to a firm's number of employees. However, for a representative firm we may assume that

(19) $$H = sN ,$$

where s denotes the average separation rate for the group of firms considered.

It follows from (19) and (18) that

(20) $$\alpha H + \gamma V = \alpha sN + \gamma fHT = \alpha sN + \gamma fsNT ,$$

and substituting this into (17) we find that

(21) $$C = \left(w + g + s\left(\alpha + \gamma fT\right)\right) N ,$$

and hence that

(22) $$p = (1+m)\left(w + g + s\left(\alpha + \gamma fT\right)\right)/a \text{ and } N = D(p)/a \text{ if } D(p) < k ,$$

where p denotes the price the firm sets (and m is a non-negative mark-up) and $D(p)$ denotes its sales at this price, while a denotes the firm's labour productivity and k its capacity. And if

---

[6] But even if we can ignore unfilled jobs when estimating the indirect effect of recruitment costs on employment, we cannot ignore the fact that unfilled jobs reduce employment directly by creating a gap between labour demand (desired employment) and employment. The measurement of unfilled jobs as distinct from job vacancies is consequently an important problem, but it is not addressed in this paper, which focuses on labour demand, not *unmet* labour demand.

sales are restricted by capacity, then production and employment are also restricted by capacity, implying that recruitment cost have no impact on employment unless (22) applies – which consequently is the only case we have to consider. Note that with a separation rate equal to 36 per cent of employment per year we have $s = 0.03$ per month, so for recruitment costs to have a noticeable impact on the product price according to (22), they have to be rather large compared to the monthly wage (*w*) and other direct costs (as measured by *g*).

7. Conclusions

To sum up, unless production and employment are restricted by capacity (*k*), price (*p*) and employment (*N*) in a firm are determined by

(23) $$p = (1+m)c = (1+m)(1+h)(w/a) \text{ and } N = D(p)/a \text{ if } D(p) < k,$$

where *w* is the wage level, *a* labour productivity, *c* marginal cost, *h* variable costs other than labour costs as a share of labour costs, $D(p)$ the firm's sales at the price it sets, and the mark-up *m* equals 0 for a firm in a competitive market and $1/(\eta - 1)$ for a monopoly or, more precisely, $m = e/(1-e)$, where $e = (p^m - c)/p^m$ and p^m solves $(p-c)/p = 1/\eta(p)$, where $\eta(p) = -pD'(p)/D(p)$.

Now, even if I have only discussed monopoly and perfect competition in this paper, (23) must be true in general for a non-negative mark-up, at least for firms with positive profits. But mark-ups are not necessarily set independently by every firm. For example, in a market with price leadership all firms but one take the market price as given, and then the mark-up for a price taker is determined by the market price *p* set by the price leader and the marginal cost *c* of the price taker, $m = (p-c)/c$.

Moreover, irrespective of the market form and the precise pricing process, trade cannot start until prices have been set by firms. Production and employment are then adjusted by a firm to the sales determined by the firm's customers – as long as production is restricted by sales and not capacity at the price set by the firm.

There is, of course, a negative relation between nominal wages and worker-hours for a firm and its industry. However, as shown in this paper, this is not because of a declining marginal product of labour, but because higher wages raise product prices and reduce sales, production, and employment. In fact, the main effect of a declining marginal product of labour is to raise product prices during a boom when production is restricted by capacity and not sales.

Moreover, unless a firm’s production and employment are restricted by its capacity, price and employment in a firm are determined by (23). Thus, a firm’s labour demand depends on product demand, capacity, labour productivity, wages and other direct costs, but also on the mark-up on direct costs chosen by the firm.

In practice there are, of course, some complications. Production does not adjust perfectly to sales unless sales precede production (production to orders). In markets where production precedes sale, as in most consumer markets, production will in general differ from sales, even if changes in inventories are negligible. And adjustment costs will stabilize employment when sales are variable or hard to predict. But this does not change the basic message of this paper, namely that a firm’s employment depends on the demand for its products at the prices it sets.

References

Borjas, George J. 2008. Labor Economics, Fourth Edition, McGrawHill.

Blinder, Alan S., Elie R. Canetti, David E. Lebow, and Jeremy B. Rudd 1998. *Asking About Prices. A New Approach to Understanding Price Stickiness*, Russel Sage Foundation.

Cahuc, Pierre and André Zylberberg 2004. *Labor Economics*, MIT Press.

Davis, Steven J., R. Jason Faberman, and John C. Haltiwanger 2013. “The Establishment-Level Behavior of Vacancies and Hiring.” *Quarterly Journal of Economics* 128: 581-622.

Hamermesh, Daniel S. 1993. *Labor Demand*, Princeton University Press.

Layard, Richard, Stephen Nickell and Richard Jackman 1991. *Unemployment. Macroeconomic Performance and the Labour Market*, Oxford University Press.

Lindbeck, Assar 1998. “New Keynesianism and Aggregate Economic Activity”, *Economic Journal*, 108, 167-180.

Marshall, Alfred 1982. *Principles of Economics*, Eighth Edition, Macmillan (first printed in 1920).

Nickell, Stephen J. 1986. “Dynamic Models of Labour Demand”. In Ashenfelter, Orley, and Richard Layard (eds.). *Handbook of Labor Economics*. North-Holland.

Pissarides, Christopher A. 1990. *Equilibrium Unemployment Theory*, Basil Blackwell.